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**IMAGE RECONSTRUCTION BASED  
MODELING OF 3D TEXTILE  
COMPOSITE (POSTPRINT)**

**Eric Zhou, David Mollenhauer, and Endel Iarve**



**MARCH 2007**

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# Image Reconstruction Based Modeling of 3D Textile Composite

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## ABSTRACT

Innovative weaving and braiding processes open up a new opportunity for making 3-D textile composites that give significantly damage-tolerant structural response with design flexibility for durable joints, near-net shape processing, etc. To fully understand the mechanical behavior of 3-D textile composites, it is essential to perform analyses to predict effective material properties and damage initiation and growth.

In this paper we present a new approach to generating 3D textile composite geometric models based on image processing techniques. The main objectives are to visualize, manipulate, and reconstruct textile internal structures based on multidimensional image data for the purpose of further mechanics analysis. A software code called the ImageScan, is developed to generate geometry models from a set of image slices of a textile composite based on image reconstruction technology. The images from an optical microscope or other source can be segmented into objective constituents and reconstructed into 3D geometry, which can be input into an appropriate mechanics model to predict the material properties and mechanical deformation under a specific boundary condition and loadings.

Key words: Composite Modeling, Image reconstruction.

## 1. INTRODUCTION

Highly accurate yarn geometry is critical for computing detailed composite performance, such as the local failure mechanism. The failure mechanism of 3D textile composites depends on the architecture of fiber reinforcement. Irregularity of tow geometry and undulation has a modest effect on the average elastic module but a strong effect on the strength [1]. The strength, notch sensitivity, and delamination resistance requires detailed modeling of tow architecture. In recent years, the image reconstruction method has widely been used in the medical industry and is able to produce 3D descriptions of various feature such as tissues and organs [2,3,4]. However, the applications in the material engineering are very few, especially in the textile composite field. However, there are some technical challenges as well. During the image reconstruction process, the internal tow geometry can be experimentally described using a variety of imaging techniques such as optical microscopy, ultrasonic imaging, and computed tomography (CT). The general image reconstruction practice in the medical field easily classifies tissues into categories with a simple grayscale range for each tissue. However in textile composites, the resin is everywhere inside of the composite and fibers often display roughly the same grayscale regardless of imaging direction with respect to fiber direction. This represents a big challenge for applying image reconstruction of 3D textile composites. Furthermore, to single out each tow, which contains thousands fibers surrounded by resin, is still an unsolved problem for image based modeling. Therefore, we investigated image based modeling techniques and began to develop a tool, which can improve the image quality, detect the yarn edge, remove the noise, reconstruct the yarn surface, and generate a 3D mesh ready model for the traditional finite element analysis

(FEA) and the novel B-spline analysis method (BSAM) developed in-house [5].

The process of this approach is in three steps; image quality improvement, image segmentation, and image reconstruction.

## **2. IMAGE IMPROVEMENT**

### **2.1 Noise removal**

The purpose of the first step is detecting the edge pixels of objects as accurately as possible by applying some image quality improvement algorithms such as Gaussian, and Histogram Equalization. The Gaussian smoothing operator is used to “blur” images and remove detail and noise while a Histogram Equalization is used to enhance the contrast of image intensity. At the beginning, a total of 16 slice images of a carbon fiber composite were taken with an optical microscope. After initial testing using ImageScan, we found that the quality of image plays a big role in the image based modeling. Noise, color intensity, and contrast will affect the modeling process. Therefore, the Gaussian algorithm was implemented into the programming code to smooth the image and reduce the noise while histogram equalization enhanced contrast to obtain a uniform image. In 2-D, an isotropic (*i.e.* circularly symmetric) Gaussian has the form:

$$G(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

This distribution is shown in Figure 1.

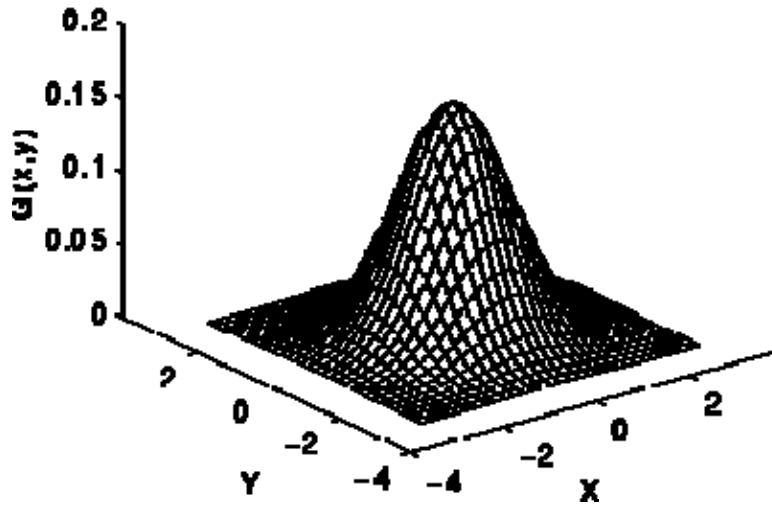


Figure1. 2-D Gaussian distribution with mean (0, 0) and  $\sigma=1$

The goal of Gaussian smoothing is to produce a discrete approximation to the Gaussian function.

## 2.2 Contrast enhancement

The histogram is constructed from a frequency table and is applicable to quantitative data. To construct a frequency table, we begin by dividing the image into an arbitrary number of subintervals. The intervals are shown on the X-axis and the number of scores in each interval is represented by the height of a rectangle located above the interval (Fig. 2). Then we redistribute intensity distributions. This technique can be used on a whole image or just on a part of the image. Currently, we are only focused on the whole image.

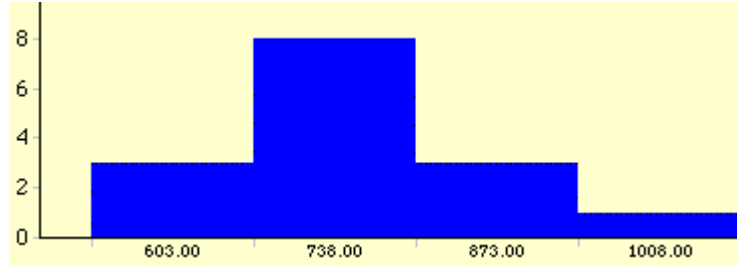


Figure 2. A Histogram

### 3. SEGMENTATION

#### 3.1 Introduction

Partitioning of an image into several constituent components is called segmentation. Segmentation is an important part of image reconstruction practice. Segmentation should be able to distinguish each object from the others and from the background. For intensity images, there are some popular approaches [6-11]: threshold techniques, edge-based methods, region-based techniques, and active contour models. Threshold techniques, which make decisions based on the local pixel information, may not handle the blurred object boundary very well. Edge-based methods can automatically detect the edges of an object. However, connecting the edge pixels is the big challenge. Region-based methods partition the image into connected regions by grouping neighboring pixels of similar intensity levels together. The adjacent regions are merged under some criterion. Therefore, over-merging could occur. The main idea of the active contour model is to start with some initial boundary shape and modify it by applying various shrink/expansion operations according the minimum energy function. In this paper, the Sobel algorithm, an edge-based method, was implemented to detect the boundary of the object, while a graph search algorithm was developed to establish the



connectivity of edge. Fully automated segmentation is still quite challenging for most applications due to the wide variety of image modalities and object properties. Unlike applications in the medical industry, image reconstruction application in textile composite does not deal with uniform objects such as tissues and organs, which likely have similar intensity levels and can be segmented with existing techniques directly. Instead, we are more interested in extracting the tow architecture from the textile composite and the tow itself consists of thousands fibers and resin, which means that the tow is not a uniform solid and the intensity of a tow is not uniform either. Thus, how to segment tows from adjacent tows becomes the central problem in the image reconstruction of textile composite.

### **3.2 Yarn classification**

In CT X-ray tomography, the image grayscale is determined by the material properties. With the grayscale range for each material, the voxels of the image can be mapped onto corresponding categories. For example, we can use a grayscale range to represent the fibers and use different grayscale to represent the resin. As we know, each tow consists of two materials, fibers and resin. Therefore, the description of a tow is not clear in the image. Existing fully automated segmentation techniques may not be able to handle this situation. However, edge-based approach may be able to detect most borderlines of a tow. With some contour tracing help from the user, we can always manage to obtain the borderlines with high precision.

### 3.3 Edge detection

The simple and fast Sobel algorithm was implemented in the program to detect the yarn boundary by calculating color intensities in all the neighbor points. It is used to find the approximate absolute gradient magnitude at each pixel in grayscale. For a 2D image, the Sobel uses a pair of 3x3 convolution masks, one calculating the gradient in the columns and the other calculating the gradient in the rows, as shown in Fig.3. These masks are designed to be applied separately to the input image, to produce separate results in each orientation, and to be combined together to find the absolute magnitude of the gradient and the orientation at each pixel. Noise removal is achieved by two methods; isolated pixel removal and short chain removal. In the first method, any point that is isolated can be removed. In the second method, any chain of edge that is short can also be removed. So far, the software can handle a variety of image formats, such as Bitmap, JPEG, GIF, TIFF, etc. It can detect the yarn boundaries by scanning the image, remove the noise, which came either from the original image, image processing, or from software conversion, and link the edges using a region weight judgment method. Fig.4 shows a result of applying Sobel edge detector and noise removal.

-1	0	+1
-2	0	+2
-1	0	+1

Gx

+1	+2	+1
0	0	0
-1	-2	-1

Gy

Figure 3. Sobel edge detector

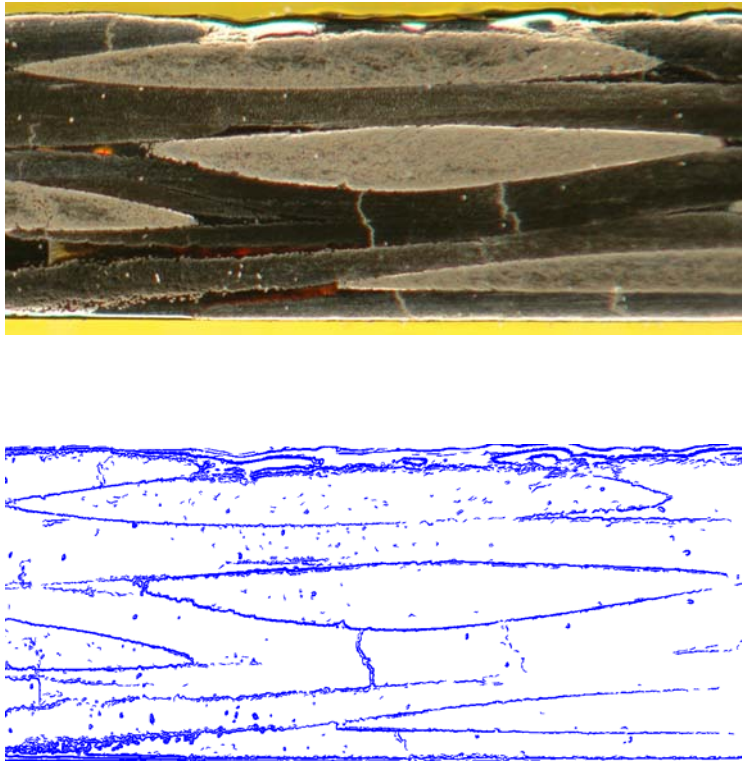


Figure 4. Result of edge detect

### **3.4 Edge connection**

Sobel detection is used to find the pixels which are located on the borderlines of the object. The concept of image segmentation is to use these edge points to construct the borderlines and use these borderlines as primitives to obtain the region segments, which, in turn, is used to reconstruct the 3D solid model of tows in textile composites. Segmentation procedures frequently result in binary format, “1” edge pixel and “0”

others. The raster scanning process of the binary permits the edge connectivity result. The procedure of edge connecting is:

- 1) First, scan each pixel and its 8 neighboring pixels for a mark. If there is no mark, then mark this pixel with a new mark representing the new edge.
- 2) If there is a mark, there two possibilities; center pixel or others. Both need the weight checking algorithm to decide whether it is edge or just noise.
- 3) If a center pixel is marked and others are edge pixels judged by the weight checking algorithm, then those pixels belong to the same edge and mark those pixels with the same value as the center pixel.
- 4) If the pixel other than the center pixel is marked, a decision has to be made whether it belongs to the previous edge or a new edge by applying the weight checking algorithm again.

After scanning the whole image, each edge pixel has been marked with edge number or noise. The pixels, which have the same edge number, are grouped together into the borderline. If the borderline is too short, we can treat it as isolated noise and remove it from the edge data. The real borderline of the object is most likely broken into several sections along the object because the image quality and constituency of image contrast. We can use the computer mouse to pick edge segments on the borderline to link them together. The region, which is contained inside the borderline, will be marked as the cross section of an individual tow.

### 3.5 Edge smooth

Since the yarn geometries are directly generated from scanning the image in great detail, the yarn edges may not be smooth enough to be used for the next modeling procedure. Therefore, edge smoothing is required to improve the quality of the image. What the Sobel algorithm found is a set of edge points on the border of a tow cross-section. So it is necessary to divide this set of points into two groups: upper bound and lower bound. In order to obtain the upper bound points and lower bound points, an  $m^{th}$  degree regression algorithm, the least square model, was written and added to the ImageScan to determine the center line of a tow. Therefore, any points, which are located on the upper half of the center line, belong to the upper bound while the other points below the center line belong to the lower bound. The Bezier polynomial was chosen to perform the edge smoothing because of its low-order feature, which would give us more flexibility. The advantages of staying with low-order polynomials include reduced computational time, greater stability and local control of shape. The Bezier polynomial does not have first-derivative continuity at its endpoints; therefore, we may only use the center segment to represent this part of a curve to improve the continuity of the curve.

$M^{th}$  degree polynomial:

$$y = a_0 + a_1x + a_2x^2 + \dots + a_mx^m$$

to approximate the given set of data,  $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ , where  $n \geq m+1$ ,

the best fitting curve  $f(x)$  has the least square error, i.e.,

$$\Pi = \sum_{i=1}^N [y_i - f(x_i)]^2 = \sum_{i=1}^N [y_i - (a_0 + a_1 x_i + a_2 x_i^2 + \dots + a_m x_i^m)]^2 = \min.$$

Bezier polynomial function:

$$P(t) = (1-t)^3 p_0 + 3t(1-t)^2 p_1 + 3t^2(1-t) p_2 + t^3$$

Fig. 5 shows the result after applying the Bezier polynomial to smooth the edge of the yarn.

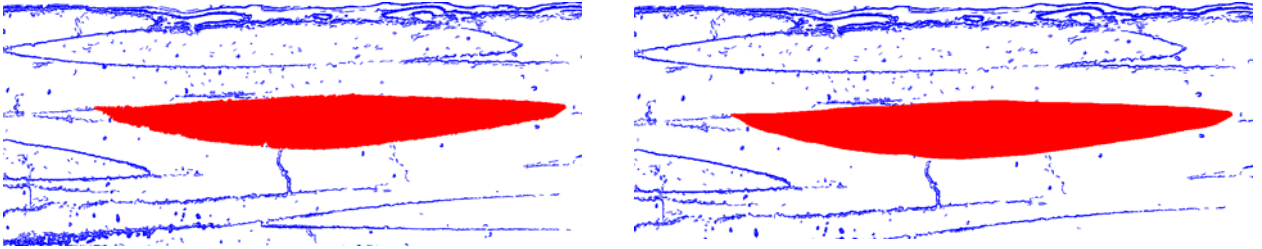


Figure 5. Applying the Bezier polynomial

### 3.5 Region classification

The cross section is the encapsulated area by the borderline, as shown in Fig. 6. In our software, each cross section of tow is classified with a unique color index. The cross sections from all the slides, which belong to the same tow, will have the same color index. The color index of the cross sections from the different tow will be different. This method can classify each cross section in all the slides to which it belongs.

## **4. RECONSTRUCTION**

The images are derived from slice data from a variety of imaging devices. The two dimensional slice data is used as input for the three dimensional reconstructions. Slices are taken at regular intervals throughout the object. Each slice is first segmented to separate the various objects. In textile composites, those objects are tows, which consist of thousands fibers and infused resin. The surface extracting algorithms are then used to create a three dimensional representation of the tow structures. In this step, we can either apply Stoke's Theorem or use a direct interpolation method to reconstruct the tow surface.

### **4.1 The Stoke's theorem approach**

The Stoke's theorem approach is in three steps: 1) the surface points and their vectors are collected and splatted into a voxel grid without needing the adjacency relations between the surface points. 2) The voxel grid is convolved with an integration filter, the fast Fourier Transform. 3) The reconstructed surface is extracted using the Marching Cubes algorithm. The tool used in this project was developed by M. Kazhdan [12]. This tool can reconstruct the surface by approximating integration using only the surface points and their normals. The advantages of this tool are that only the surface points and their normals are needed, and the result is a water-tight solid model, and points on the surface are not necessarily uniformly distributed. In the test run, a total of 16 slices were obtained by an optical microscope. The result is very good when the gap between subsequent slices is small. When the gap was increased by a factor 2.5, the result is rough, as shown in Fig.6. There are two methods available to solve this problem. The

first one is to obtain finer slice spacing. For non-destructive image acquisition processes, such as CT, this method will be simple and easy. However, it will be catastrophic if the image spacing were too large for a set of serial sectioned images, because the sample has been destroyed during the imaging process. Considering the possibility of missing or incomplete image data, the second method, the interpolating method, becomes essential in image reconstruction of textile tows. There are many approaches to interpolating a curved surface. These include linear, bi-cubic polynomial, and spline patches. Bezier patches are smoother than most. Although splines give even more smoothness, the complexity of splines makes Bezier patches more appealing in practice. We approach the Bezier surface by applying the blending functions, as shown as below, which blends the data at 16 control points. Furthermore, the uniform continuous surface of tow geometry has been achieved by modifying the Bezier patches.

$$\mathbf{B}(u,v) = \sum_{i=0}^{N_i} \sum_{j=0}^{N_j} p_{i,j} \frac{N_i!}{i! (N_i - i)!} u^i (1 - u)^{N_i - i} \frac{N_j!}{j! (N_j - j)!} v^j (1 - v)^{N_j - j}$$

$$\begin{matrix} 0 \leq u \leq 1 \\ 0 \leq v \leq 1 \end{matrix}$$



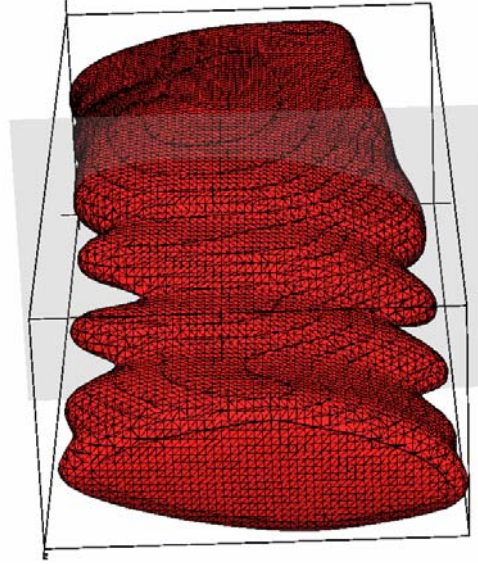


Figure 6. Reconstruction from Stoke's theorem

#### 4.2 Direct interpolation method

Stokes' theorem generates the surface definition of a solid tow without any information regarding the tow path. As we know, the material property of a fiber tow responds anisotropically inside the textile composite. The tow path is an extremely important parameter for numerical analysis. Therefore, extra effort has been made to convert the tow geometry to a mechanics model favorable definition, in which the tow is represented as a series of cross sections along the tow axis. For most fiber reinforced composites, tows are reinforced in three directions, X, Y and Z. In our software, all the points are stored in a cubic grid in the XYZ coordinate system after scanning a series of images. Since we have defined the tow as a series of cross sections along the tow axis, not necessary perpendicular to the tow axis, therefore, a cross section of a tow can be simply defined as a set of points, which is in a plane. This plane is a cross section of 3D

grid. From these cross sections, we can obtain the tow information. For example, for 2D woven, we can have the warp cross section directly from XYZ grid in the X direction and the filler cross section data in the Y direction. For most textile composites, such as 2D triaxial braided, 3D braided, and 3D ply-to-ply woven, tows are reinforced along X, Y, and Z direction. Therefore, a more effective method, the direct interpolating method was developed to reconstruct the tow geometry. The procedure is a simple 2-step process. The first step is to redistribute borderline points of a tow in each slice uniformly. The next step is to create piecewise facets accordantly, as shown in Fig.7. Fig.8 shows images that were taken by an optical microscope. A total of 50 slices were gathered in this case. Fig. 9 shows the results after segmentation. The color represents the identity of the tow. Each tow has the same color index. The 3D solid model generated using the image reconstruction method is shown in Fig. 10. Both Stokes' theorem approach and the interpolating approach are used in this case.

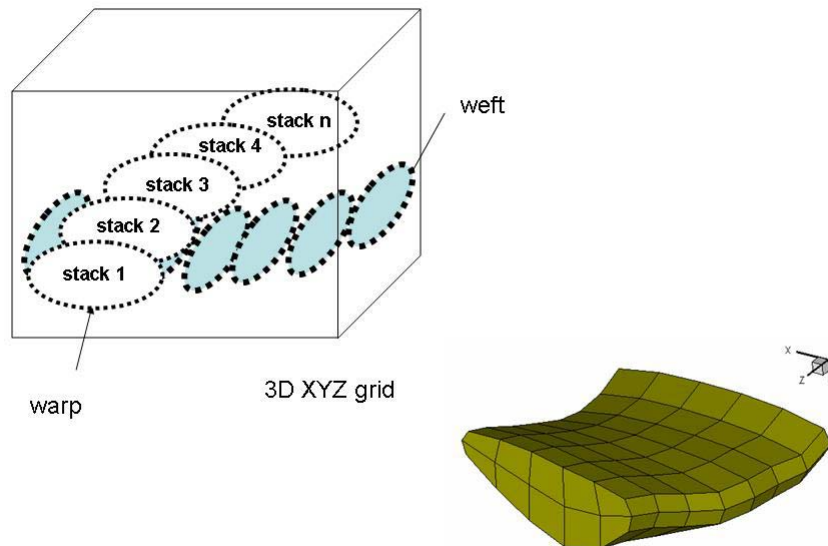


Figure 7. Direct interpolation method



Figure 8. 50 slices of image from an optical microscope

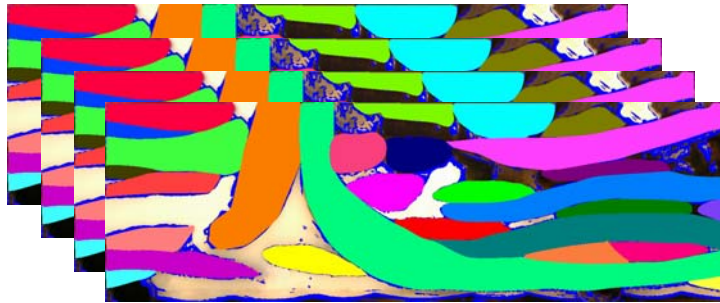


Figure 9. After segmentation

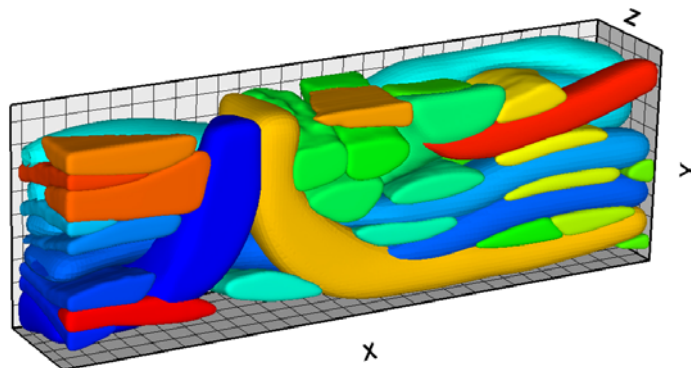


Figure 10. Image reconstruction result

## **5. FUTURE WORK**

As we know, segmentation is the central problem of image reconstruction. It is used to single out the object of interest from others and from the background. Fully automated segmentation is still quite a challenge for textile composite applications due to the wide variety of image modalities and object properties. It is difficult to extract the tow architecture from the image because the tow itself consists of thousands fibers and resin, which means that the tow is not a uniform solid and the intensity of a tow is not uniform either. Thus, how to group some individual fibers into several tows effectively becomes the central problem in image reconstruction. In the current approach, 80% of the total time was spent on image segmentation. In the future, more effective segmentation methods will be investigated.

## **6. SUMMARY**

Highly accurate yarn geometry is critical to predict composite performance. Because of the variable nature of textiles, it becomes necessary to use information directly from the textile composite of interest. The internal tow geometry can be experimentally described using a variety of imaging techniques, such as optical microscopy, ultrasonic imaging, and computed tomography (CT). An image based geometry reconstruction method was presented. It can improve the image quality, detect the tow edge, remove the noise, reconstruct the yarn surface, and generate a 3D mesh ready model for numerical modeling. The image reconstruction based modeling method consists of three steps: image improvement, segmentation, and reconstruction. The Gaussian algorithm was implemented into the software to smooth the image and reduce

the noise while histogram equalization was used to enhance the image contrast. An edge-based segmentation approach, which detects the yarn boundary by applying the Sobel algorithm, was used. Then a raster scanning process of the binary is performed to identify the edge pixels which belong to the same edge and connect them into the borderline of the tow. Finally the segmented region is defined by its borderline. Edge smoothing is performed using a Bezier curve patch. There are two methods of image reconstruction used in this project, the Stoke's theorem and direct interpolating method. The Stoke's theorem takes an oriented point set as input and returns a solid, water-tight model. This method reconstructs the surface by approximating integration using only the surface points and their normals. The advantages of this approach are that only the surface points and their normals are needed. The disadvantage is the extra effort that has to be made to convert the tow geometry generated by the Stoke's theorem into a numerical model favorable format. Since most fiber reinforced composite, the tows are reinforced along the X, Y, Z directions, we can easily obtain the cross section of tow directly from the XYZ grid. Therefore, the direct interpolating method could be a more effective method to reconstruct geometry from the image. The demonstration of a textile fabric reconstruction was made by using image reconstruction method with both Stokes' theorem and the direct interpolating approach.

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